A Literature Review on Applications of Graph Theory in Various Fields

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***Abstract*: Graph theory is widely used to prove many mathematical theorems and models.This paper present the various applications and techniques of graph theory to solve problems in different fields of science and technology in addition to mathematics. These fields include computer science, chemistry, biology, digital image processing, website designing, software engineering and operations research. The present work is focus on the history, introduction, terminology, interpretation of graph and its applications in the various fields of science and technology.**

***Keywords*: Bipartite graph, Euler graph, Hamiltonian graph, connected graph, graph colouring, tree**

Introduction:
Graph theory is nothing but a branch of Discrete Mathematics. Graph Theory is the study of graphs which are mathematical structures not only used in computer science but in many fields. Two problem areas are mainly considered. Problems such as classical problems and problems from an application. The classic problem is defined with the help of graph theory as connectivity, cut, path and flow, cololoring problems and theoretical aspects of graph drawing. The problems from an application focus on experimental research and implementation of graphs theory algorithms [6]. Graphs are important because graph is a way of expressing information in pictorial form. A graph shows information that equivalent to many words. Many problems that are considered difficult to determine or implement can easily be solved by graphic theory. There are many types of graphs as part of graph theory. Each type of graph is related to a particular property. One of these graphs is used in many applications for troubleshooting. Because of the representation power of graphs and flexibility many problem can be represented and solved easily [1].

History:

The origin of graph theory started in 1735 with the problem of Koinsberg Bridge. Euler studied the problem of Koinsberg Bridge and constructed a structure to solve the problem; this structure is called as Eulerian graph. The details complete graph and bipartite graph is given by A.F Mobius In 1840, Kuratowski proved that they are planar by means of recreational problems. The concept of tree was implemented by Gustav Kirchhoff in 1845, which are used in the calculation of currents in electrical networks or circuits. Thomas Gutherie found the four colour problem in 1852. William Hamilton and P. Kirkman discussed the cycles on polyhydra then invented the concept named Hamiltonian graph ; In 1913 H.Dudeney mentioned a puzzle problem. In 1878 Sylvester introduced the term "Graph". in 1941 Ramsey worked on the concept called colorations which lead to the identification of another branch of graph theory called extremel graph theory .The four colour problem was solved using computers by Heinrich In 1969[4].

Basics:
Before we can understand application of graphs we need to know some definitions that are part of graphs theory.

Graph: A graph is denoted as G(V,E).A graph consists of set of vertices V and set of edges E[1].

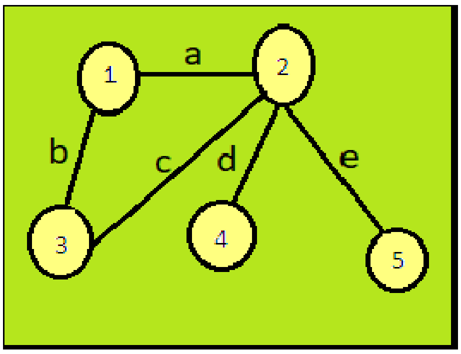

Figure 1: Simple Graph
Figure: 1 shows the simple graph with five vertices,V={1,2,3,4,5} and edge set E={(1,3),(1,2),(2,3),(2,4),(2,5)}.

Vertex: The vertex is the point at which two edges meet [1].

Figure: 1 shows five vertices 1,2,3,4 and 5.

Edge: An edge is a line by which two vertices are connected with each other, denoted by e= (v, u).
Figure: 1 shows five edges a, b, c, d and e

Directed Graph: A directed graph is a set of vertices connected by edges, with each vertex having a direction associated with it.Edges are usually represented by arrows in directed graph [7].

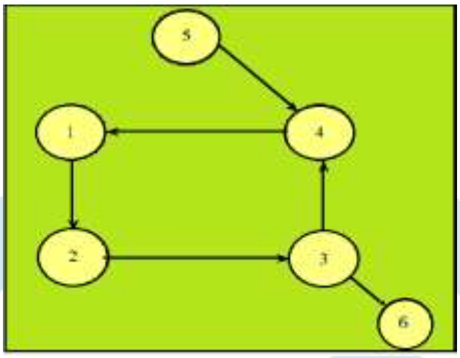

Figure 2: Directed Graph

Undirected Graph: In an undirected graph the edges are bidirectional, there is no direction associated with an edge. So, the graph can be traversed in either direction [7].

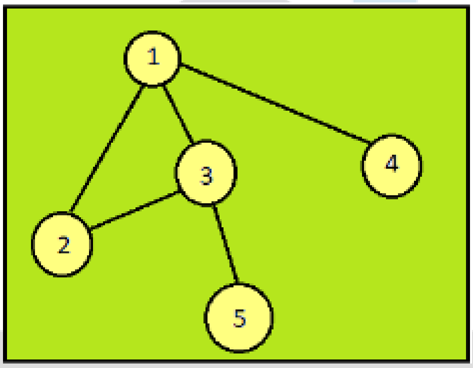

Figure 3: Undirected Graph

Connected graph: A graph G= (V, E) is called as a connected graph if there exists a path between every pair of vertices in graph G.

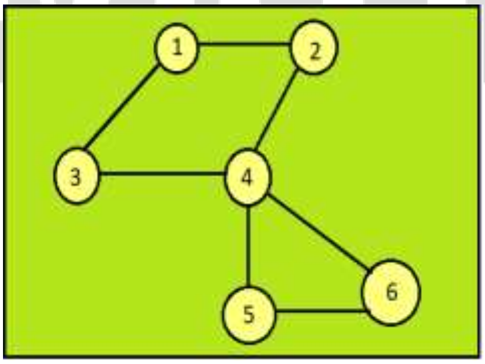

Figure 4: Connected Graph

Loop: When an edge is drawn from an edge to itself is called as a loop.

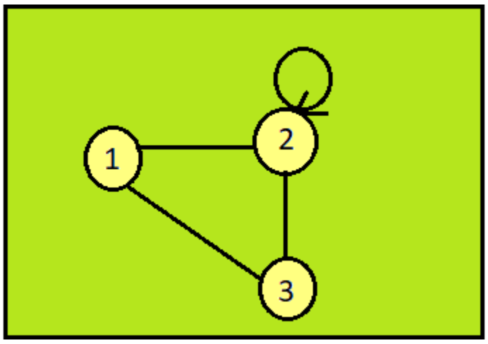

Figure 5: Loop

Figure: 5 indicates node 2 has self loop.

Parallel edges: In a graph G= (V, E) when a vertex is connected to other vertex by more than one edge then the edge is called parallel edge. The graph is called multigraph.

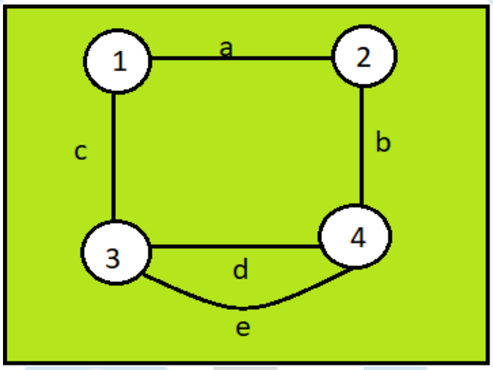

Figure 6: Parallel Edge

Figure: 6 shows that, edges d and e are parallel edges.

Adjacent vertices: In a graph G= (V, E) two vertices are said to be adjacent when there is an edge between them.
Figure: 6, shows vertex 1 and vertex 3 are adjacent.

Degree of a vertex: Degree of the vertex is the number of edges that are incident to the vertex.
Figure: 6 shows, degree of vertex 4 is 3.

Regular graph: The graph G = (V,G) is called as regular graph if all vertices of graph have same degree.

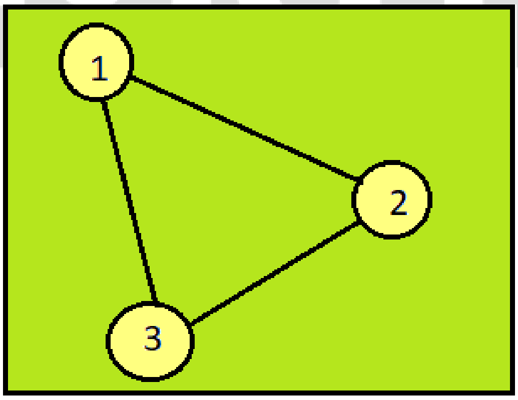

Figure 7: Regular Graph

Figure: 7 shows Regular Graph with 3 vertices. All vertices having same degree.

Complete graph: A graph G= (V, E) is called as complete graph if every vertex has an edge to all other vertices.

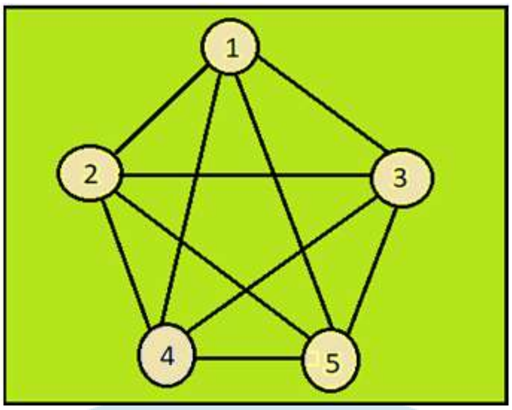

Figure 8: Connected Graph

Figure: 8 shows complete graph on 5 vertices, each vertex is adjacent to every other vertex.

Cyclic graph: If there is cycle in a graph then a graph G= (V, E) is called cyclic graph.

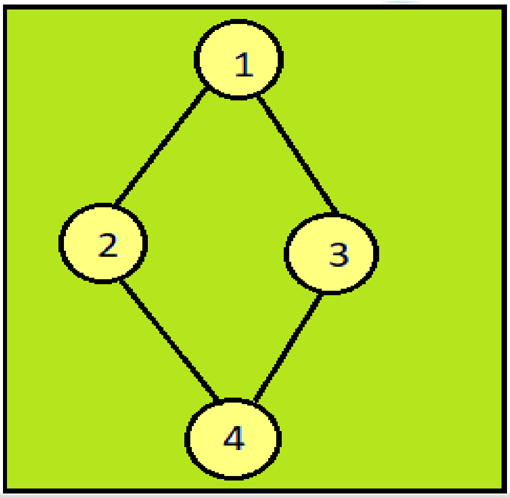

Figure 9: cyclic graph

Figure: 9 shows cyclic graph on four vertices.

Acyclic graph: and if there is no any cycle exist then a graph G= (V, E) is called acyclic graph.

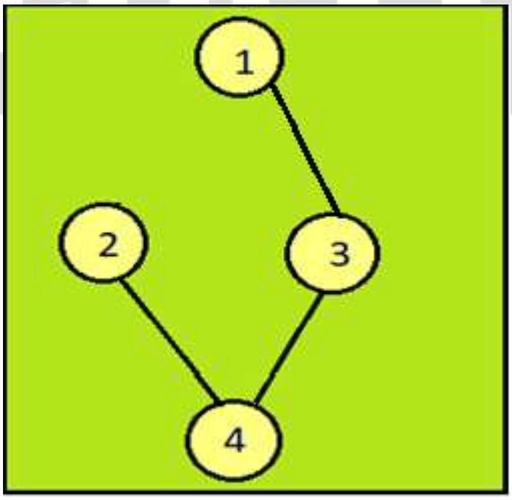

Figure 10: acyclic graph

Tree: A tree is a connected graph in which there is no any cycle.

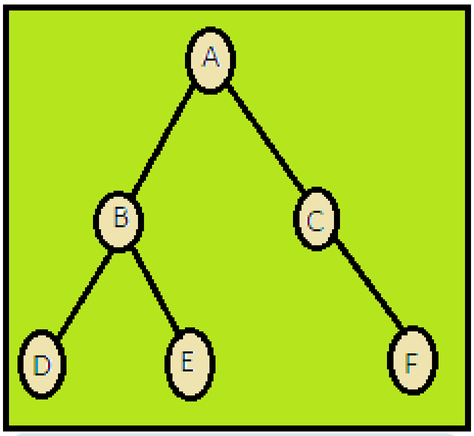

Figure 11: Tree

Figure: 11 shows Tree having six nodes, A is a root node, B & C are internal nodes and D, E, F are leaf nodes.

Bipartite graph: A simple graph G= (V, E) is called a bipartite graph if with vertex partition V= {V1, V2} each edge of graph G will join a vertex in V1 to a vertex in V2.

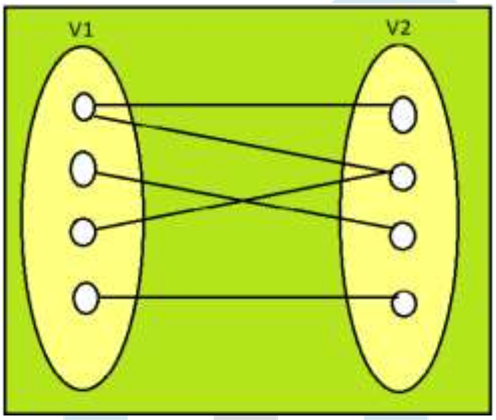

Figure 12: Bipartite Graph

Complete bipartite graph: A simple graph G= (V, E) is called a complete bipartite graph if with vertex partition V= {V1, V2} every vertex in V1 will join every vertex in V2.

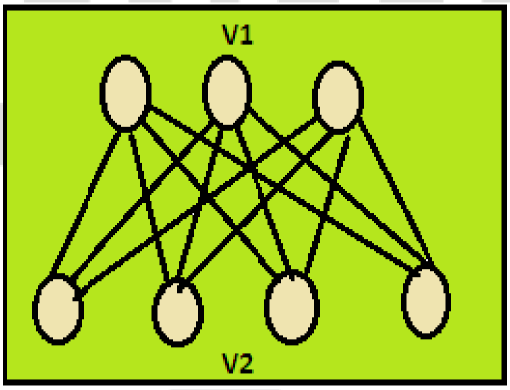

Figure 13: Complete Bipartite Graph

Figure 13 shows complete bipartite graph in which every vertex of set V1 is adjacent to vertex of set V2.

Vertex coloring: Assigning colours to the vertices of a graph G in a way that no two adjacent vertices of G will have same colour.

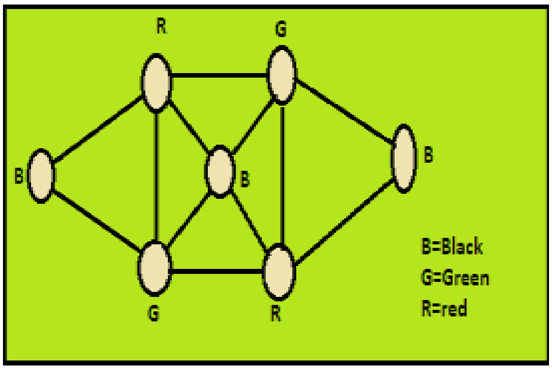

Figure 14: Vertex colouring

Chromatic number: chromatic number of graph is the minimum number of colours which we need for coloring vertices of a graph G.

Spanning tree: Spanning tree is the subset of graph which covers all the vertices of graph G. And it should not form a circuit.

Cut vertex: A vertex V of a graph G is a cut vertex if removal of it produces a disconnected graph.

Euler graph: Euler graph is a connected graph G = (V, E) if there is if there is a closed trail which includes every edge of the graph G [8].

Euler Path: A path that uses every edge of a graph exactly once that starts and ends at different vertices called Euler path [8].

Euler Circuit - An Euler circuit is a circuit that uses every edge of a graph exactly once and it always starts and ends at the same vertex. An Euler graph is one in which all vertices are of even degree [8].

Hamiltonian graph: A connected graph G=(V, E) is said to be Hamiltonian graph, Hamiltonian graph is a connected graph if there is a cycle which includes every vertex of graph and the cycle formed is called Hamiltonian cycle.

Application:

Computer Networks

In today's life computer networks are extremely popular, where nodes are connected to each other via specific links. This final network of nodes forms a graph. Graph is used to form a network of nodes in computer network and enable efficient packet routing. This may includes finding the shortest paths between nodes, analyze the current network traffic and find fastest root, finding cost efficient route. Algorithms such as Dijkstra’s algorithm and Bellman-Ford algorithms are used to in the various ways with graph to find the solutions [1].

Digital Image Processing

Image Analysis mainly performed on digital image processing techniques is the methodology by which information from images is extracted. Using a graph theoretic approach the image processing techniques can be improved. The applications of using graphs in image processing are: to find edge boundaries using graph search algorithms in segmentation.
• To calculate the alignment of the picture.
• Finds mathematical constraints such as entropy by using minimum spanning tree.
• Finding distance transforms of the pixels and calculates the distance between the interior pixels by using shortest path algorithms [6].

Operating System

Various practical problems can be solved with the help of graph in the field of operating system such as job scheduling and resource allocation problems. Job Scheduling is the process of allocating system resources to many different tasks by an operating system [2]. The graph colouring concept can be applied in job scheduling problems of CPU, jobs are assumed as vertices of the graph and there will be an edge between two jobs and there will be one to one relationship between feasible scheduling of graphs.
Graph purpose in operating system:
• System processes are represented in graph form.
• Powerful tool for graph like-query
• Graph databases are used for associative data sets that map more directly to the structure of object-oriented applications [5].

Making Schedule or Time Table

Suppose we want to make a university exam schedule. We have a list of different subjects and students in each subject. Most of the subjects were general students (of the same batch, some backlog students, etc.). Since no two exams are scheduled at the same time with normal students, how do we schedule the exam? What is the minimum time slot required to schedule all exams? This problem can be represented as a graph where each vertex is a subject and the mean between the two vertices is a normal pupil. So this is a graphic colour problem where the minimum number of time slots is equal to the number of colours in the graph [5].

Map Coloring

Geographical maps of countries or states can also be created using graph colouring technique where two adjacent cities cannot be assigned same colour. Four colours are sufficient to colour any map [5].

Software Engineering

Graph has many applications in software engineering. For example: between the requirements specification, data flow diagrams are used where vertices represent transformation and edges indicate data flow. In the design phase, graphical design is used to describe the relationships between the modules. During the test, the control flow of the program related to the measurement of the complexity of the McCabe to which graphs are applied in the order of execution instructions and so on. Even software process management has network diagram applications that contain graph algorithms [5].

Chemistry

In model molecule structures for computer processing graphs are used. Atoms are considered as vertices of a graph and the bonds that connects them are edges between them. Based on the properties of compounds this structures are created and then taken for analysis and processing. It can be used to study the structure of molecules and to check similarity level between molecules [1].

Biology

Graph is also used for the analysis in biological networks. In biology analysis the number of components of the system and their interactions is distinguish as network and they are normally represented as a graph where lots of nodes are connected with thousands of vertices [6]. Graph theory is widely used in following biological analysis; Protein-protein interaction (PPI) networks, Regulatory networks (GRNs), Signal transduction networks, and Metabolic and biochemical networks. When we analysis these different components then it will be generated the structure network which is similar to one of the graph component in graph theory. Graph isomorphism method can be used for matching two components [1].

Web Page Designing

A graph is useful in website designing. Where the web pages are represented by vertices and the hyper links are represented by edges in the graph. This concept is called as web graph. Other application of graphs is in web community. In which the vertices represent classes of objects. Each vertex which represents a type of object is connected to every vertex which represents other kind of objects. In graph theory such a graph is a complete bipartite graph. Using graph representation in website development we get:
• Searching and community discovery.
• Graph representation in web site utility evaluation and link structure.
• finds all connected component and provide easy detection.

Operational Research

Many important OR problems are solved using graphs. A Graph is used to model the transportation of commodity from one place to another. A network called transport network. The main objective is to maximize the flow and minimize the cost within the prescribed flow [4].

Conclusion:

In this paper different definition of various graphs is described. This paper is also providing the basic idea of graphs. All the required terminologies of graph theory are covered by these definitions. Different applications of graph theory has been identified and divided as per their fields. This paper will explains where different graphs of graph theory are used in these real world applications. This paper helps to the students for getting the deep knowledge of graph theory and its relevance with different subjects like operating systems, Networks, Databases, software engineering, Biology, Chemistry, Operation Research and Digital Image Processing etc.

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